

## Research Paper

## Single Acceptance Sampling Plan Based on Truncated-life Tests for the Exponentiated Moment Exponential Distribution With Application in Bladder Cancer Data

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## ABSTRACT

**Background and Purpose:** In statistical quality control, inspecting complete products is very hard due to its cost and time. This research uses a time-truncated single acceptance sampling plan to investigate bladder cancer data for the exponentiated moment exponential distribution.

**Materials and Methods:** A single acceptance sampling plan is used to investigate bladder cancer data when a patient's remission times following an exponentiated moment exponential distribution. Minimum sample sizes, operating characteristic function, and the associated producer's risks are obtained and calculated.

**Results:** Minimum sample sizes necessary to ensure a certain mean lifetime for selected acceptance numbers and consumer confidence levels are obtained. The operating characteristic function and the associated producer's risks are provided. We also analyzed the minimum ratios of the mean life to the specified life.

**Conclusion:** The results show optimal sample sizes decrease when the time to pre-determined mean lifetime ratio increases. Besides, the operating characteristic values of the proposed single sampling plan increase when the ratios of the mean life to the specified life increase.

**Keywords:** Bladder cancer, Remission, Sample size, Urinary bladder neoplasms

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## 1. Introduction

Quality control is one of the important applications of the statistical inference method. In this regard, the two important methods are acceptance sampling and control charts. Acceptance sampling is one of the applications of statistical testing in quality control. There are two types of acceptance sampling plans: attributes acceptance sampling plan and variable acceptance sampling plan. In attributes acceptance sampling plans, the decision is based on the number of failures of defectives. In contrast, a variable acceptance sampling plan is used for deciding based on measurements.

A time-truncated sampling plan checks whether a product can be accepted. Due to several constraints like time and cost, checking all products is hardly possible or impossible. Thus, a random sample is drawn from the lot. The test is finished, and the decision is made by a pre-specified time  $t$ . Let  $P^*$  be the consumer's confidence level, and then the goal is to find a confidence limit on the mean life of products,  $\mu$ , and to set a specified mean life,  $\mu_0$  with a probability of at least  $P^*$ .

The single acceptance sampling plan is the most used attribute acceptance sampling plan, which is simple to use. In this paper, we focus on this type of plan. In a single sampling plan, a lot is accepted if and only if the observed number of failures does not become bigger than  $c$ . One can end the test whenever the number of failures gets more than  $c$  before the time  $t$  and decide to reject the lot. A single sampling plan includes the following parameters:

- (i) the number of items on test  $n$ ,
- (ii) the acceptance number  $c$ ,
- (iii) the ratio  $t/\mu_0$

Figure 1 shows the operation of a single acceptance sampling plan. This plan has been proposed for many lifetime distributions, for example, the Marshall-Olkin extended Lomax model [1], the generalized exponential model [2], the half-normal model [3], and the exponentiated Fréchet model [4]. Recently, some studies discussed the single sampling plan for the weighted exponential, extended exponential, and power Lomax models [5-7].

This paper provides a single acceptance sampling plan when the lifetime distribution follows an exponentiated moment exponential (EME) distribution. The EME model was introduced by [8] with the following probability density function (pdf) (Equation 1):

$$1. f(t, \alpha, \beta) = \frac{\alpha}{\beta^2} t e^{-\frac{t}{\beta}} \left[ 1 - \left( 1 + \frac{t}{\beta} \right) e^{-\frac{t}{\beta}} \right]^{\alpha-1}, \quad t > 0,$$

And the corresponding cumulative density function is given by (Equation 2):

$$2. F(t, \alpha, \beta) = \left[ 1 - \left( 1 + \frac{t}{\beta} \right) e^{-\frac{t}{\beta}} \right]^{\alpha}, \quad t > 0,$$

Where  $\alpha > 0$  and  $\beta > 0$  are the shape and scale parameters, respectively. The EME distribution with parameters  $\alpha$  and  $\beta$  is denoted as EMED ( $\alpha, \beta$ ). The  $r^{\text{th}}$  moment of the EME distribution is given by (Equation 3):

$$3. E(T^r) = \alpha \beta^r I(r, \alpha),$$

where (Equation 4):

$$4. I(r, \alpha) = \int_0^{\infty} [1 - (1+t)e^{-t}]^{\alpha-1} t^{r+1} e^{-t} dt.$$

Thus, the mean of the EME distribution is given by (Equation 5):

$$5. \mu = \alpha \beta I(1, \alpha).$$

## 2. Materials and Methods

### Materials

#### Real data

A real data set regarding the remission times of a random sample of 30 bladder cancer patients (in months) was considered and analyzed. This data set was previously examined in a study [9].

#### The model used

We assume a parametric model for the lifetime distribution to design a sampling plan. An EME distribution with pdf (1) is used for modeling the bladder cancer data.

#### Method

Suppose the lifetime follows an EMED ( $\alpha, \beta$ ), where  $\alpha$  is known. Let  $\mu$  denote the true average lifetime of a product, and then a lot is considered good if  $\mu \geq \mu_0$ , otherwise, it is considered a bad lot. Thus, the consumer's risk is the probability of accepting a bad lot, whereas the

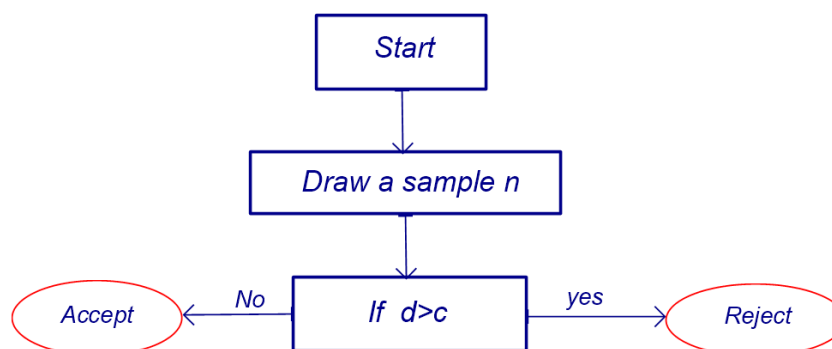


Figure 1. Operation for the single acceptance sampling plan

producer's risk is the probability of rejecting a good one. In this study, we fixed the consumer's risk to become not greater than  $P^*$ , where  $0 \leq P^* \leq 1$ . We suppose that the size of the lot is so large that we can use the binomial distribution in our work. The acceptance or rejection of the lot corresponds to the acceptance or rejection of the hypothesis  $\mu \geq \mu_0$ . From (3), we can say that the hypothesis  $\mu \geq \mu_0$  corresponds to  $\beta \geq \beta_0$  where  $\beta_0 = \mu_0/m$  and  $m = \alpha(1, \alpha)$ . The plan is to set a specified time,  $t$  and is characterized by the triplet  $(n, c, t/\mu_0)$ , including the number of items  $n$  to be drawn from the lot, the acceptance number  $c$  and the ratio  $t/\mu_0$ .

### 3. Results

#### Minimum sample size

We estimate the minimum sample size  $n$  satisfying the consumer's risk, the probability of accepting a bad lot, is not to exceed  $1 - P^*$ , when  $\mu = \mu_0$ . The probability of accepting a lot is given by (Equation 6):

$$6. \sum_{i=0}^n \binom{n}{i} p^i (1-p)^{n-i}$$

Where  $p = F(t, \alpha, \beta)$  = probability of a failure before time  $t$  that is given by  $p = [1 - (1 + md)e^{-md}]$ , where  $d = t/\mu$ . For  $\mu = \mu_0$ , we have  $p_0 = [1 - (1 + md_0)e^{-md_0}]^\alpha$ , where  $d_0 = t/\mu_0$ . Therefore, the required  $n$  is the smallest positive integer that satisfies the following inequality (Equation 7).

$$7. \sum_{i=0}^n \binom{n}{i} p_0^i (1-p_0)^{n-i} \leq 1 - P^*$$

The minimum values of  $n$  satisfying (4) are computed and summarized in Tables 1 and 2 for  $\alpha = 0.5, 2$ ,  $P^* = 0.90, 0.95, 0.99$ , and  $d_0 = 0.4, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5, 3.0$ . Suppose that the remission times (in months) of bladder cancer patients follow the EME distribution with shape parameter  $\alpha = 0.5$ , and we wish to establish that the mean lifetime is at least 1.39 months with a prob-

ability  $P^* = 0.95$ . The life test is terminated at  $t = 2.08$  months. Therefore, from Table 1, for  $d_0 = 1.5$  and  $c = 3$ , the minimum sample size is 8, namely 8 patients should be put on test. If no more than 3 patients fail during 2.08 hours, then the experimenter can assert that the true-mean of remission times  $\mu$  is at least 1.38 months with a confidence level of 0.95.

The shape of the required minimum sample size versus  $t/\mu_0$  for  $\alpha = 2$ ,  $c = 3$ , and selected values of the confidence level  $P^*$  is plotted in Figure 2.

#### Operating characteristic function

The operating characteristic (OC) function is the probability of accepting a lot. A sampling plan is preferable if its OCs approach more rapidly to one. For the proposed sampling plan  $(n, c, t/\mu_0)$ , the OC is defined as (Equation 8):

$$8. OC = \sum_{i=0}^n \binom{n}{i} p^i (1-p)^{n-i}$$

Note that increasing the ratio  $\mu/\mu_0$  will increase the acceptance probability. Tables 3 and 4 provide the OC function values for the EME distribution adopted from Tables 1 and 2 for  $c = 3$ , various values of  $P^*$ ,  $\mu/\mu_0$  and  $\alpha = 0.5, 2$ , respectively. Besides, the values of OC versus  $\mu/\mu_0$  are plotted in Figure 3 for  $\alpha = 0.5$ ,  $t/\mu_0 = 0.4$  and  $c = 3$ . From Tables 3 and 4, we observe that the OC function values  $\mu/\mu_0$  increase as the ratio increases. If we consider  $P^* = 0.95$ ,  $d_0 = 0.6$ ,  $c = 3$  and  $\mu/\mu_0 = 8$ , then the sample size is 15 (Table 3), and the OC value is 0.9834. This outcome reveals that the lot is accepted if less than or equal to 3 patients out of 15 patients fail before time point  $t = 0.83$  months and  $\mu \geq 8t/0.6 = 11.07$  months, then the lot will be accepted with a probability of at least 0.9834.

**Table 1.** Minimum sample size tested for a time  $t$  to assert with probability  $P^*$  and acceptance number  $c$  when  $\mu \geq \mu_0$  ( $\alpha=0.5$ )

$P^*$	$c$	0.4	0.6	0.8	1	1.5	2	2.5	3
0.90	0	6	4	3	3	2	2	1	1
	1	11	8	6	5	4	3	3	2
	2	15	11	8	7	5	4	4	4
	3	19	14	11	9	7	6	5	5
	4	23	16	13	11	8	7	6	6
	5	27	19	15	13	10	8	7	7
	6	31	22	17	15	11	10	9	8
	7	35	25	20	17	13	11	10	9
	8	39	27	22	18	14	12	11	10
0.95	0	8	6	4	4	3	2	2	1
	1	13	9	7	6	4	3	3	3
	2	18	12	10	8	6	5	4	4
	3	22	15	12	10	8	6	6	5
	4	26	18	14	12	9	8	7	6
	5	30	21	17	14	11	9	8	7
	6	34	24	19	16	12	10	9	8
	7	38	27	21	18	14	11	10	10
	8	42	30	24	20	15	13	12	11
0.99	0	12	8	6	5	4	3	2	2
	1	18	12	10	8	6	4	4	3
	2	23	16	12	10	7	6	5	5
	3	28	19	15	12	9	7	6	6
	4	33	23	18	15	11	9	8	7
	5	37	26	20	17	12	10	9	8
	6	42	29	23	19	14	12	10	9
	7	46	32	25	21	15	13	11	10
	8	50	35	27	23	17	14	13	12

**Table 2.** Minimum sample size tested for a time t to assert with probability P\* and acceptance number c when  $\mu \geq \mu_0$  ( $\alpha=2$ )

P*	c	0.4	0.6	0.8	1	1.5	2	2.5	3
0.90	0	25	9	5	3	2	1	1	1
	1	42	15	8	6	3	3	2	2
	2	58	21	11	8	5	4	3	3
	3	72	26	15	10	6	5	4	4
	4	87	32	17	12	7	6	5	5
	5	101	37	20	14	9	7	6	6
	6	114	42	23	16	10	8	8	7
	7	128	47	26	18	11	9	9	8
	8	141	52	29	20	13	10	10	9
0.95	0	32	11	6	4	2	2	1	1
	1	51	18	10	7	4	3	2	2
	2	68	24	13	9	5	4	3	3
	3	84	30	16	11	7	5	5	4
	4	99	36	20	13	8	6	6	5
	5	114	41	23	15	9	7	7	6
	6	128	47	26	18	11	9	8	7
	7	143	52	29	20	12	10	9	8
	8	157	57	32	22	13	11	10	9
0.99	0	49	17	9	6	3	2	2	1
	1	71	25	13	9	5	3	3	2
	2	90	32	17	11	6	5	4	4
	3	108	38	21	14	8	6	5	5
	4	125	45	24	16	9	7	6	6
	5	141	51	27	18	11	8	7	7
	6	157	56	31	21	12	9	8	8
	7	173	62	34	23	13	11	9	9
	8	188	68	37	25	15	12	11	10

**Table 3.** Operating characteristic values of the sampling plan for  $c=3$ ,  $\alpha=0.5$  and selected values of  $P^*$ 

$P^*$	$n$	$d_0$	2	4	6	8	10	12
0.90	19	0.4	0.5730	0.9122	0.9737	0.9897	0.9952	0.9975
	14	0.6	0.5219	0.8946	0.9675	0.9871	0.9939	0.9968
	11	0.8	0.5137	0.8916	0.9664	0.9866	0.9937	0.9967
	9	1.0	0.5273	0.8966	0.9682	0.9874	0.9941	0.9969
	7	1.5	0.4631	0.8709	0.9588	0.9834	0.9921	0.9958
	6	2.0	0.4071	0.8444	0.9485	0.9788	0.9898	0.9945
	5	2.5	0.4563	0.8652	0.9563	0.9822	0.9915	0.9955
	5	3.0	0.3177	0.7906	0.9259	0.9683	0.9844	0.9915
0.95	22	0.4	0.4518	0.8661	0.9570	0.9826	0.9917	0.9956
	15	0.6	0.4619	0.8707	0.9588	0.9834	0.9921	0.9958
	12	0.8	0.4346	0.8588	0.9543	0.9814	0.9911	0.9953
	10	1.0	0.4276	0.8556	0.9531	0.9809	0.9909	0.9951
	8	1.5	0.3270	0.8016	0.9313	0.9710	0.9858	0.9923
	6	2.0	0.4071	0.8444	0.9485	0.9788	0.9898	0.9945
	6	2.5	0.2420	0.7389	0.9031	0.9575	0.9788	0.9883
	5	3.0	0.3177	0.7906	0.9259	0.9683	0.9844	0.9915
0.99	28	0.4	0.2573	0.7529	0.9098	0.9608	0.9806	0.9893
	19	0.6	0.2636	0.7582	0.9122	0.9620	0.9812	0.9897
	15	0.8	0.2443	0.7432	0.9055	0.9588	0.9795	0.9887
	12	1.0	0.2650	0.7596	0.9130	0.9624	0.9814	0.9898
	9	1.5	0.2218	0.7242	0.8966	0.9545	0.9772	0.9874
	7	2.0	0.2458	0.7437	0.9056	0.9588	0.9795	0.9887
	6	2.5	0.2420	0.7389	0.9031	0.9575	0.9788	0.9883
	6	3.0	0.1341	0.6240	0.8444	0.9276	0.9625	0.9788

**Minimum ratio of  $\mu/\mu_0$  for the acceptability of a lot**

For a specified producer's risk and a sampling plan  $(n, c, t/\mu_0)$ , one may seek the value of quality level  $\mu/\mu_0$  that will ensure the producer's risk does not become greater than  $\gamma$ . Recall that the producer's risk is the probability of rejection of a lot when it is good, i.e.  $\mu \geq \mu_0$  or equivalently  $\beta \geq \beta_0$ . Then, we seek the minimum values of  $\mu/\mu_0$  that satisfy (Equation 9).

$$9. \sum_{i=c+1}^n \binom{n}{i} p_0^i (1-p_0)^{n-i} \leq \gamma.$$

The minimum values of  $\mu/\mu_0$  satisfying (6) are presented in Tables 5 and 6 with the producer's risk of  $\gamma=0.05$  and  $\alpha=0.5$ , 2. Consider a situation where one intends to establish the mean life  $\mu_0$  is at least 1.39 months with a probability  $P^*=0.95$  and desires to end the life test at  $t=0.83$  months. If the actual mean life is  $\mu$  about 8.33 hours (namely, the value of  $\mu/\mu_0$  is around 6), then from Table 5 with the producer's risk 0.05, we find  $c=3$ . We also perceive that the required  $n$  in Table 1 corresponds to the values of  $P^*=0.95$ ,  $c=3$ , and  $d_0=0.6$  is 15. Hence, a sampling plan  $(n, c, t/\mu_0)=(15, 3, 0.6)$  may be taken.

**Table 4.** Operating characteristic values of the sampling plan for  $c=3$ ,  $\alpha=2$  and selected values of  $P^*$ 

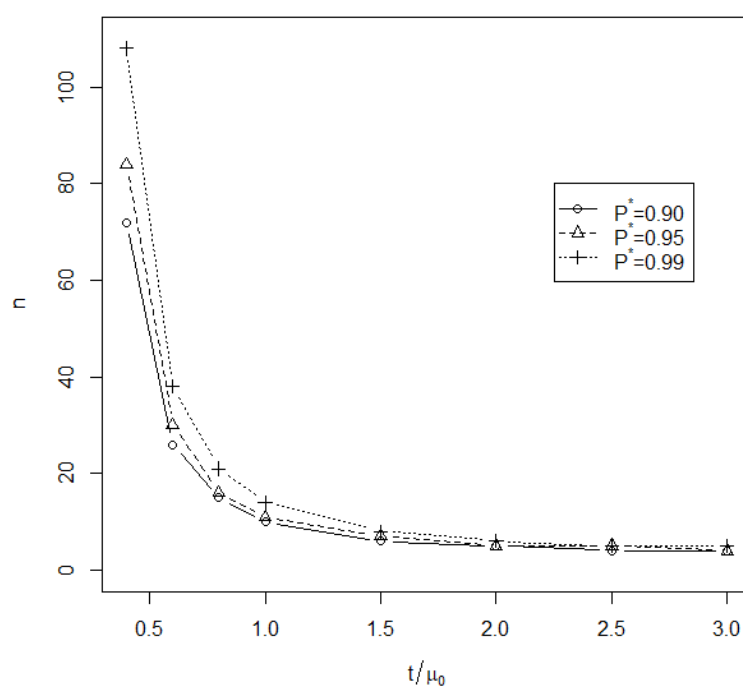
$P^*$	$n$	$d_0$	2	4	6	8	10	12
0.90	72	0.4	0.9912	1.0000	1.0000	1.0000	1	1
	26	0.6	0.9809	1.0000	1.0000	1.0000	1	1
	15	0.8	0.9592	1.0000	1.0000	1.0000	1	1
	10	1.0	0.9391	0.9999	1.0000	1.0000	1	1
	6	1.5	0.8561	0.9996	1.0000	1.0000	1	1
	5	2.0	0.7000	0.9972	1.0000	1.0000	1	1
	4	2.5	0.7081	0.9952	0.9999	1.0000	1	1
	4	3.0	0.4993	0.9807	0.9994	1.0000	1	1
0.95	84	0.4	0.9851	1.0000	1.0000	1.0000	1	1
	30	0.6	0.9692	1.0000	1.0000	1.0000	1	1
	16	0.8	0.9494	1.0000	1.0000	1.0000	1	1
	11	1.0	0.9161	0.9999	1.0000	1.0000	1	1
	7	1.5	0.7608	0.9990	1.0000	1.0000	1	1
	5	2.0	0.7000	0.9972	1.0000	1.0000	1	1
	5	2.5	0.3987	0.9812	0.9995	1.0000	1	1
	4	3.0	0.4993	0.9807	0.9994	1.0000	1	1
0.99	108	0.4	0.9665	1.0000	1.0000	1.0000	1	1
	38	0.6	0.9353	1.0000	1.0000	1.0000	1	1
	21	0.8	0.8836	0.9999	1.0000	1.0000	1	1
	14	1.0	0.8270	0.9998	1.0000	1.0000	1	1
	8	1.5	0.6561	0.9982	1.0000	1.0000	1	1
	6	2.0	0.5012	0.9926	0.9999	1.0000	1	1
	5	2.5	0.3987	0.9812	0.9995	1.0000	1	1
	5	3.0	0.1812	0.9322	0.9972	0.9998	1	1

### Bladder cancer data

In this section, we analyze a real data set from a study [9] regarding the remission times of a random sample of 30 bladder cancer patients (in months):

0.08,0.20,0.40,0.50,0.51,0.81,0.90,1.05,1.19,1.26,1.35,1.40,1.46,1.76,2.02,2.02,2.07,2.09,2.23,2.26,2.46,2.54,2.62,2.64,2.69,2.69,2.75,2.83,2.87,3.02.

We perform a formal Kolmogorov-Smirnov (K-S) test to check if the EME distribution fits the above data well. The K-S statistic is computed as  $D=0.20217$  with the corresponding  $P$  that equals 0.1496. The maximum likelihood estimates of the scale and shape parameters are calculated as  $\hat{\beta}=0.8480144$  and  $\hat{\alpha}=1.0739784$ , respectively. Thus, we conclude that the two-parameter EME distribution can perfectly model the above data. Figure 4 gives the P-P plot of data.



**Figure 2.** The minimum sample size versus  $t/\mu_0$  for  $\alpha=2$  and  $c=3$

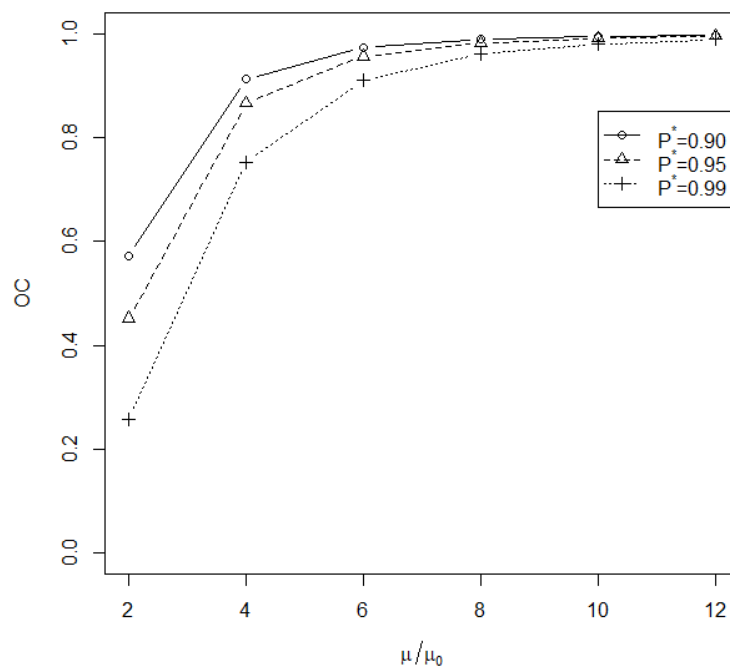
#### 4. Discussion

The mean lifetime can be estimated as  $\hat{\mu} = m\hat{\beta} = 1.76$ . If we assume that  $t=1.06$  months, then we get:

$$10. d_0 = \frac{t}{\mu_0} = \frac{1.06}{1.76} = 0.6.$$

Consider  $P^*=0.9$ , with  $\alpha=1.07$ . From Table 7, we obtain  $n=30$  and  $c=6$ . Therefore, if the number of patients

with remission time before  $t=1.06$  months is less than or equal to 6, we can accept the lot with the assured mean remission of 1.76 and a probability of 0.9. Since the number of patients with remission time before  $t=1.06$  is 8, we can reject the lot.



**Figure 3.** Operating characteristic values versus  $\mu/\mu_0$  for  $\alpha=0.5$ ,  $c=3$  and  $t/\mu_0=0.4$



**Table 5.** Minimum ratio of  $\mu/\mu_0$  for the acceptability of a lot with producer's risk of  $\gamma=0.05$  ( $\alpha=0.5$ )

<b>P*</b>	<b>c</b>	<b>0.4</b>	<b>0.6</b>	<b>0.8</b>	<b>1</b>	<b>1.5</b>	<b>2</b>	<b>2.5</b>	<b>3</b>
0.90	0	44.48	44.49	44.49	55.62	55.63	74.17	46.39	55.67
	1	11.23	12.03	11.74	11.98	13.93	13.13	16.41	11.35
	2	6.51	6.97	6.48	6.93	6.84	6.71	8.39	10.06
	3	4.87	5.21	5.27	5.17	5.63	6.06	5.74	6.88
	4	4.06	4.05	4.22	4.30	4.22	4.61	4.46	5.35
	5	3.57	3.60	3.62	3.78	3.99	3.78	3.71	4.46
	6	3.24	3.29	3.22	3.43	3.36	3.86	4.04	3.87
	7	3.01	3.08	3.13	3.19	3.30	3.38	3.57	3.46
	8	2.84	2.79	2.90	2.79	2.93	3.03	3.22	3.15
0.95	0	59.31	66.72	59.32	74.15	83.42	74.17	92.72	55.67
	1	13.38	13.63	13.89	14.67	13.93	13.13	16.41	19.69
	2	7.91	7.67	8.35	8.10	8.62	9.12	8.39	10.06
	3	5.71	5.63	5.83	5.88	6.70	6.06	7.58	6.88
	4	4.64	4.63	4.61	4.79	4.97	5.63	5.76	5.35
	5	4.01	4.03	4.21	4.15	4.55	4.55	4.72	4.46
	6	3.59	3.64	3.69	3.73	3.81	3.86	4.04	3.87
	7	3.30	3.36	3.33	3.43	3.67	3.38	3.57	4.28
	8	3.08	3.16	3.23	3.21	3.24	3.47	3.79	3.86
0.99	0	88.95	88.96	88.96	92.67	111.22	111.23	92.72	111.26
	1	18.73	18.46	20.32	20.04	22.01	18.57	23.21	19.69
	2	10.24	10.46	10.22	10.44	10.39	11.49	11.40	13.68
	3	7.38	7.31	7.51	7.28	7.76	7.50	7.58	9.09
	4	5.99	6.08	6.17	6.25	6.45	6.62	7.03	6.92
	5	5.03	5.13	5.09	5.26	5.11	5.31	5.69	5.66
	6	4.52	4.52	4.62	4.61	4.71	5.08	4.83	4.85
	7	4.07	4.08	4.10	4.16	4.04	4.40	4.23	4.28
	8	3.73	3.77	3.72	3.83	3.88	3.90	4.33	4.54

**Table 6.** Minimum ratio of  $\mu/\mu_0$  for the acceptability of a lot with producer's risk of  $\gamma=0.05$  ( $\alpha=2$ )

<b>P*</b>	<b>c</b>	<b>0.4</b>	<b>0.6</b>	<b>0.8</b>	<b>1</b>	<b>1.5</b>	<b>2</b>	<b>2.5</b>	<b>3</b>
0.90	0	3.28	3.67	4.12	4.40	5.82	6.19	7.74	9.28
	1	2.17	2.36	2.54	2.86	3.21	4.28	4.32	5.18
	2	1.86	2.00	2.11	2.32	2.79	3.29	3.36	4.03
	3	1.70	1.81	1.96	2.06	2.37	2.80	2.89	3.47
	4	1.61	1.71	1.78	1.90	2.11	2.50	2.60	3.12
	5	1.54	1.63	1.70	1.80	2.09	2.29	2.41	2.89
	6	1.49	1.57	1.64	1.72	1.94	2.14	2.68	2.72
	7	1.45	1.52	1.59	1.66	1.83	2.02	2.53	2.58
	8	1.42	1.49	1.55	1.62	1.85	1.93	2.41	2.47
0.95	0	3.51	3.89	4.35	4.81	5.82	7.76	7.74	9.28
	1	2.30	2.51	2.75	3.03	3.65	4.28	4.32	5.18
	2	1.96	2.10	2.25	2.43	2.79	3.29	3.36	4.03
	3	1.78	1.90	2.01	2.15	2.59	2.80	3.50	3.47
	4	1.67	1.78	1.90	1.97	2.29	2.50	3.12	3.12
	5	1.60	1.69	1.80	1.86	2.09	2.29	2.87	2.89
	6	1.54	1.64	1.72	1.82	2.08	2.39	2.68	2.72
	7	1.50	1.58	1.67	1.75	1.95	2.25	2.53	2.58
	8	1.47	1.54	1.62	1.70	1.85	2.14	2.41	2.47
0.99	0	3.95	4.41	4.90	5.43	6.60	7.76	9.69	9.28
	1	2.54	2.78	3.00	3.31	4.00	4.28	5.35	5.18
	2	2.13	2.31	2.48	2.64	3.06	3.72	4.11	4.93
	3	1.93	2.06	2.23	2.38	2.77	3.16	3.50	4.20
	4	1.80	1.93	2.05	2.17	2.46	2.81	3.12	3.75
	5	1.71	1.83	1.92	2.02	2.37	2.56	2.87	3.44
	6	1.65	1.74	1.86	1.96	2.20	2.39	2.68	3.21
	7	1.60	1.69	1.78	1.88	2.06	2.44	2.53	3.03
	8	1.56	1.65	1.73	1.81	2.05	2.31	2.67	2.89

**Table 7.** Minimum sample size tested for a time to assert with probability  $P^*=0.95$  and acceptance number  $c$  when  $\mu \geq \mu_0$   $\alpha=(1.07)$ 

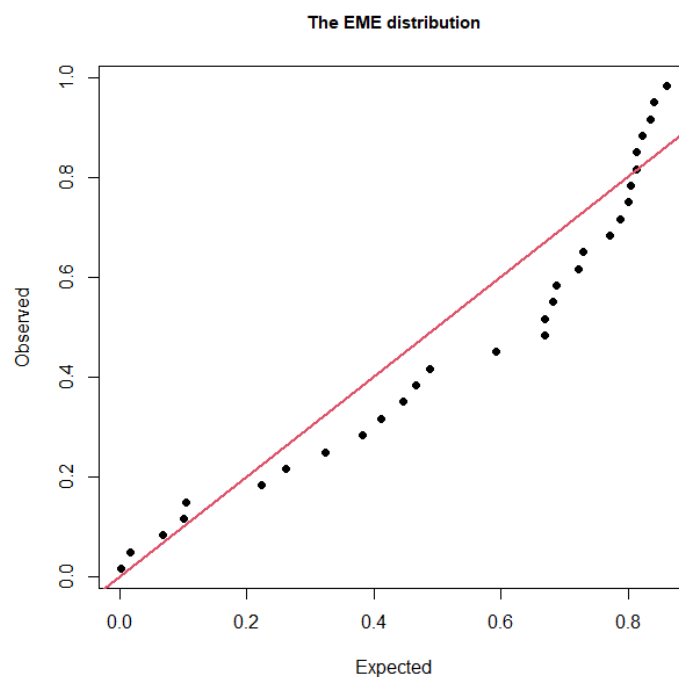
c	0.4	0.6	0.8	1	1.5	2	2.5	3
0	12	6	4	3	2	1	1	1
1	21	11	7	5	3	3	2	2
2	28	15	10	8	5	4	4	3
3	36	19	13	10	6	5	5	4
4	43	23	15	12	8	6	6	5
5	50	27	18	14	9	8	7	6
6	57	30	20	16	11	9	8	8
7	63	34	23	18	12	10	9	9
8	70	37	25	20	13	11	10	10

## 5. Conclusion

Reliability sampling plans are used to determine the acceptability of a product with respect to its lifetime. Due to various restrictions like time and cost, a complete inspection is impossible. Then, the consumer draws a random sample from the lot, and a decision (acceptance or rejection) is made at a pre-specified time.

In this paper, we developed a single acceptance sampling plan based on the truncated life test for the EME distribution. The minimum sample size required to guar-

antee a certain mean lifetime of the test items is provided. Two tables and Figure 1 are presented to report the results. A sampling plan is preferable if its OCs approach rapidly to 1. The OC function values are obtained and analyzed in some tables and Figure 1. The results show that the OC function values increase as the ratio  $\mu/\mu_0$  increases. Besides, some tables are provided for the minimum ratio of the true mean life to a specified life for the acceptability of a lot with a certain producer's risk. Moreover, a real data set regarding the remission times of a random sample of 30 bladder cancer patients is provided for illustrative purposes. The K-S test is used for

**Figure 4.** P-P plot of data

the goodness test of fit for the EME distribution, which shows that EMED (1.0739784,0.8480144) has a good fit to the data. It is noticed here that all computations of this paper were done using statistical software R version 4.2.3 [10]. There were no limitations in this study.

## Ethical Considerations

### Compliance with ethical guidelines

There were no ethical considerations to be considered in this research.

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### Authors contributions

Investigation and analyses: Zainalabideen, AL-Husseini; Initial idea, writing original draft, methodology, and investigation: Mehran Naghizadeh Qomi; Investigation and editing: Seyed Mohammad Taghi MirMostafaei.

### Conflict of interest

The authors declared no conflict of interest.

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